

Max/Min Value of Trigonometric Expression

The simplest method

- 1) Put 0° , 90° , 180° and 270° into the expression
- 2) Calculate and jot down the values
- 3) The maximum/minimum values among those numbers are the maximum/minimum values of the expression.

Principle: The maximum/minimum values of trigonometric expression usually appear at one of the four angles. **But this method does not always work.**

Expressions that must work.

For the following forms of expressions, this method **must work provided that $0^\circ \leq \theta < 360^\circ$** . It can be proved by calculus. (Not shown here)

- 1) $a \sin \theta + b$ or $a \cos \theta + b$
- 2) $\frac{a \sin \theta + b}{c \sin \theta + d}$ or $\frac{a \cos \theta + b}{c \cos \theta + d}$
- 3) $\frac{a \sin^2 \theta + b}{c \sin^2 \theta + d}$ or $\frac{a \cos^2 \theta + b}{c \cos^2 \theta + d}$ or $\frac{a \sin^2 \theta + b}{c \cos^2 \theta + d}$ or $\frac{a \cos^2 \theta + b}{c \sin^2 \theta + d}$

Note: a, b, c, d are constants

Expressions that may not work

- 1) $(a \sin \theta + b)^2 + c$ or $(a \cos \theta + b)^2 + c$
- 2) $a \sin^2 \theta + b \sin \theta + c$ or $a \cos^2 \theta + b \cos \theta + c$
- 3) $\frac{a \sin \theta + b}{c \cos \theta + d}$ or $\frac{a \cos \theta + b}{c \sin \theta + d}$

HKDSE candidates are able to handle case (1) and (2). It is explained in the following section. For case (3), it is beyond the ability of HKDSE candidates. It is difficult to handle even for students studying M2. So, case (3) can be ignored.

How to handle $(a \sin \theta + b)^2 + c$

Finding maximum value:

- 1) Considering the case that $(a \sin \theta + b)$ is positive, try to find its maximum value.
- 2) Considering the case that $(a \sin \theta + b)$ is negative, try to find its minimum value. (負最多)
- 3) Hence, obtain the maximum value of $(a \sin \theta + b)^2$

Finding minimum value:

- 1) From both positive and negative direction, try to find the value of $(a \sin \theta + b)$ closest to zero.
- 2) Hence, obtain the minimum value of $(a \sin \theta + b)^2$

Example: Find maximum and minimum value of $(2 \sin \theta - 1)^2 + 1$

<p>Max: When $\sin \theta = +1$, $(2 \sin \theta - 1)^2 = +1$ When $\sin \theta = -1$, $(2 \sin \theta - 1)^2 = +9$</p>	<p>Min: When $\sin \theta = \frac{1}{2}$, $(2 \sin \theta - 1)^2 = 0$</p>
<p>\therefore Maximum value = $9 + 1 = 10$</p>	<p>\therefore Minimum value = $0 + 1 = 1$</p>

How to handle $a \sin^2 \theta + b \sin \theta + c$

- 1) Using the method of completing the square, convert the expression into the form of $a(\sin \theta - h)^2 + k$
- 2) With reference to the section 'How to handle $(a \sin \theta + b)^2 + c$ ', find the maximum/minimum value of the expression.

三角代數式的最大/最小值

最簡單的方法

- 1) 把 $0^\circ, 90^\circ, 180^\circ$ 和 270° 代入數式
- 2) 計算數式的值並記下
- 3) 這些數字的最大和最小值就是代數式的最大和最小值

原理：三角代數式的最大值和最小值通常在這四個角度其中之一出現。

但此方法並非 100% 可行。

必定可行的代數式

以下型式的代數式，此方法必定可行(前提是 $0^\circ \leq \theta < 360^\circ$)。它們可透過微積分來證明(在此省略)。

1) $a \sin \theta + b$ 或 $a \cos \theta + b$

2) $\frac{a \sin \theta + b}{c \sin \theta + d}$ 或 $\frac{a \cos \theta + b}{c \cos \theta + d}$

3) $\frac{a \sin^2 \theta + b}{c \sin^2 \theta + d}$ 或 $\frac{a \cos^2 \theta + b}{c \cos^2 \theta + d}$ 或 $\frac{a \sin^2 \theta + b}{c \cos^2 \theta + d}$ 或 $\frac{a \cos^2 \theta + b}{c \sin^2 \theta + d}$

註： a, b, c, d 為常數

未必可行的代數式

1) $(a \sin \theta + b)^2 + c$ 或 $(a \cos \theta + b)^2 + c$

2) $a \sin^2 \theta + b \sin \theta + c$ 或 $a \cos^2 \theta + b \cos \theta + c$

3) $\frac{a \sin \theta + b}{c \cos \theta + d}$ 或 $\frac{a \cos \theta + b}{c \sin \theta + d}$

上列的情況(1) 和 (2)，文憑試考生是有足夠能力去處理，其方法將在以下章節說明。而情況(3) 是超越了中學生的能力範圍，就算 M2 學生都難以處理，因此可把它忽略。

$(a \sin \theta + b)^2 + c$ 的處理方法

求最大值:

- 1) 考慮 $(a \sin \theta + b)$ 是正數時，嘗試找它最大的數值
- 2) 考慮 $(a \sin \theta + b)$ 是負數時，嘗試找它最小的數值 (即是「負最多」)
- 3) 從而得到 $(a \sin \theta + b)^2$ 的最大值

求最小值:

- 1) 分別從正、負兩方面嘗試求取 $(a \sin \theta + b)$ 最接近零的數值
- 2) 從而得到 $(a \sin \theta + b)^2$ 的最小值

例題: 求 $(2 \sin \theta - 1)^2 + 1$ 的最大值和最小值

最大值: 當 $\sin \theta = +1$, $(2 \sin \theta - 1)^2 = +1$
 當 $\sin \theta = -1$, $(2 \sin \theta - 1)^2 = +9$

\therefore 最大值 $= 9 + 1 = 10$

最小值: 當 $\sin \theta = \frac{1}{2}$, $(2 \sin \theta - 1)^2 = 0$

\therefore 最小值 $= 0 + 1 = 1$

 $a \sin^2 \theta + b \sin \theta + c$ 的處理方法

- 1) 運用配方法，把數式變成 $a(\sin \theta - h)^2 + k$ 此型式
- 2) 參考以上《 $(a \sin \theta + b)^2 + c$ 的處理方法》章節求取數式的最大/最小值