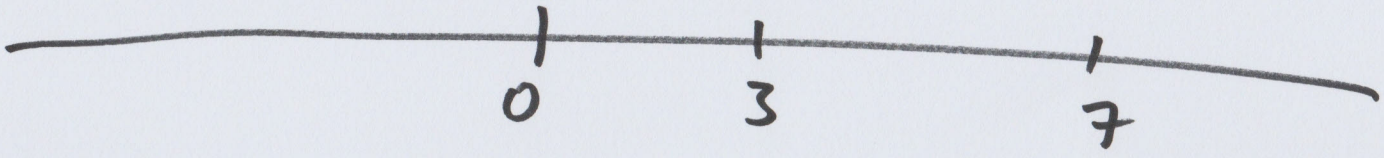
中三數學科

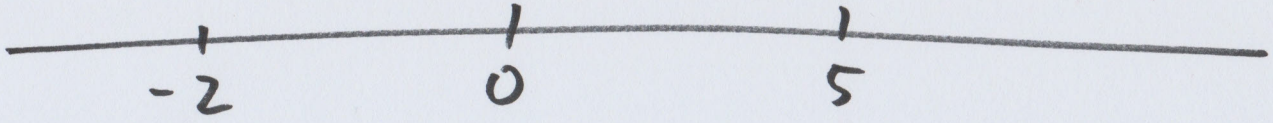
直線的坐標幾何

Coordinate Geometry of Straight Lines



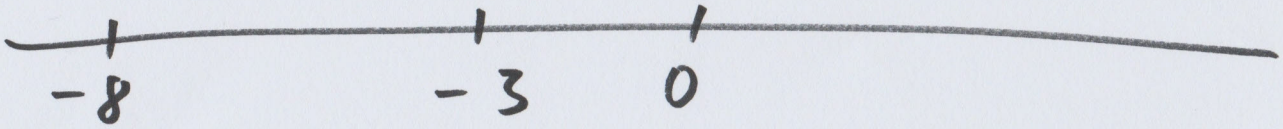
$7 - 3 = 4$

$3 - 7 = -4$



$5 - (-2) = 7$

$-2 - 5 = -7$



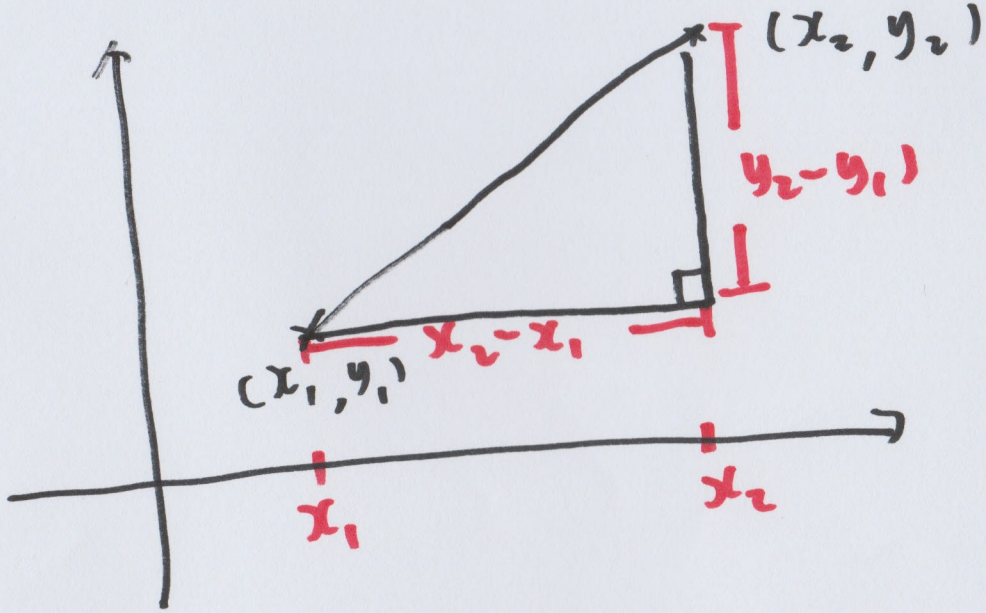
$-3 - (-8) = 5$

$-8 - (-3) = -5$

Distance Formula

$$A = (x_1, y_1) \ \& \ B(x_2, y_2)$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



$(7,2)$ $(3,10)$

$$d=\sqrt{(7-3)^2+(2-10)^2} \quad =\sqrt{80}$$

$(5,-3)$ $(-6,-4)$

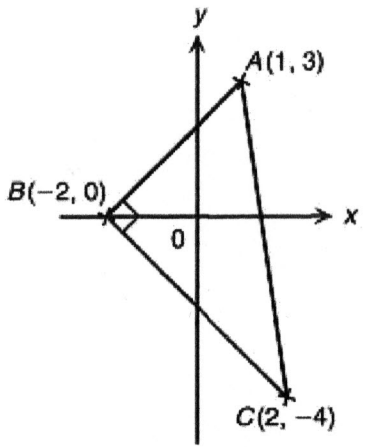
$$d=\sqrt{(5-(-6))^2+(-3-(-4))^2} \quad =\sqrt{122}$$

$$d=\sqrt{(5+6)^2+(-3+4)^2}$$

16. In the figure, $A(1, 3)$, $B(-2, 0)$ and $C(2, -4)$ are the vertices of a right-angled triangle.

(a) Find, in surd form, the lengths of AB and BC.

(b) Find the area of $\triangle ABC$.



Source: Mathematics in Action (Second Edition) Supplementary Exercises 3B

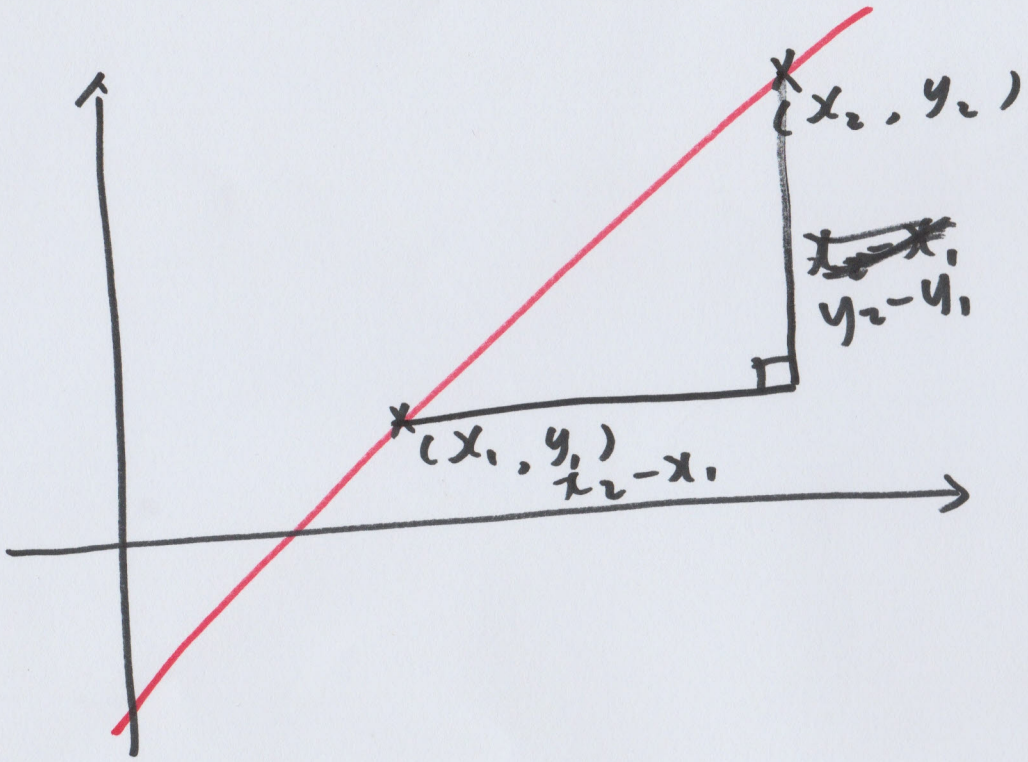
(a) $AB = \sqrt{(1+2)^2 + (3-0)^2} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(2+2)^2 + (-4-0)^2} = \sqrt{32} = 4\sqrt{2}$

(b) $\frac{3\sqrt{2} \times 4\sqrt{2}}{2}$

$= \frac{12 \times 2}{2} = 12$

Slope 斜率



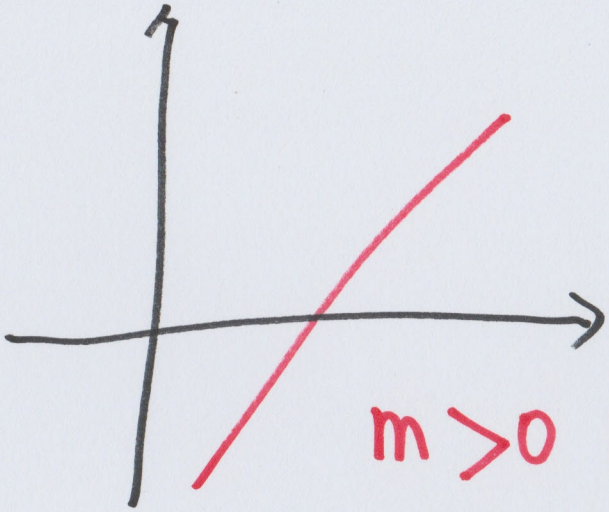
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$(7, 2)$ $(3, 10)$

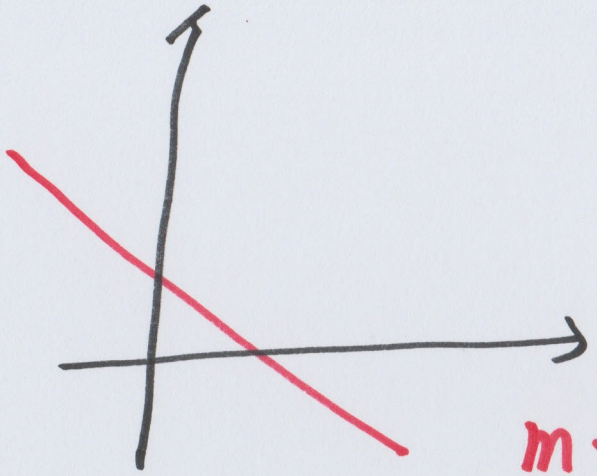
$$m = \frac{2 - 10}{7 - 3} = -2$$

$(5, -3)$ $(-6, -4)$

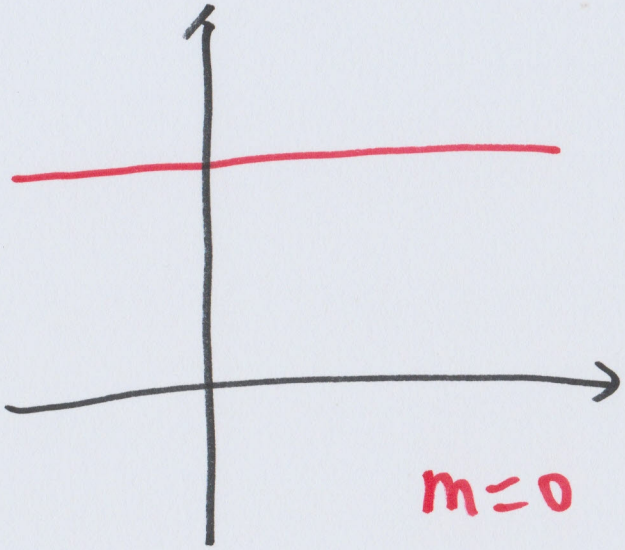
$$m = \frac{-3 + 4}{5 + 6} = \frac{1}{11}$$



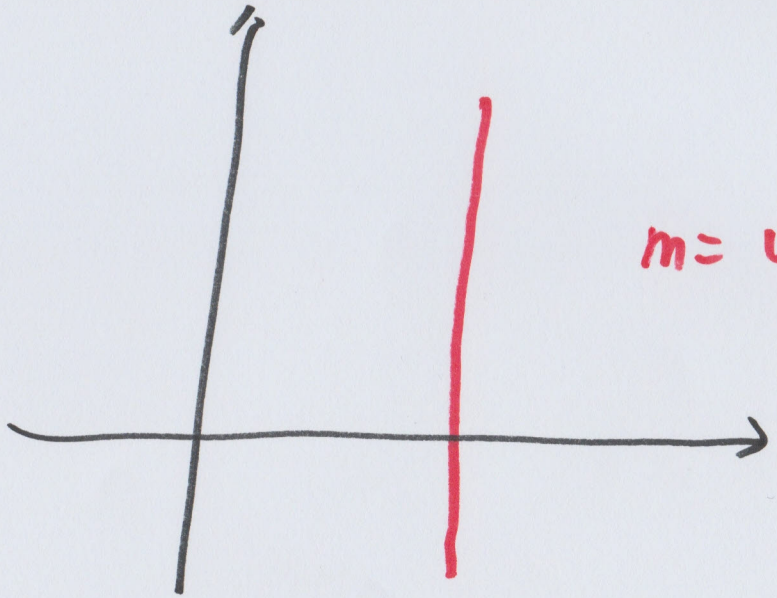
$m > 0$ (+)



$m < 0$ (−)



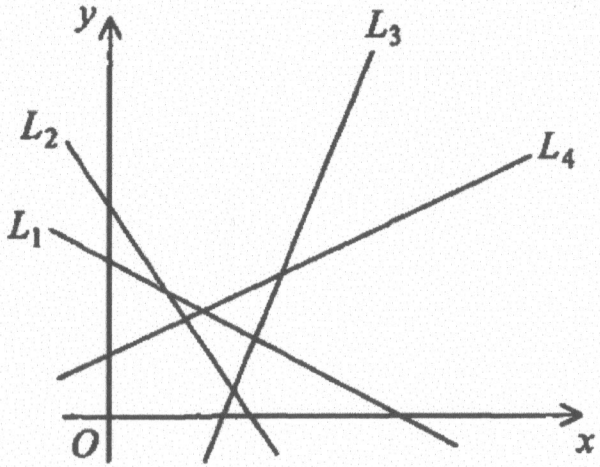
$m = 0$



$m =$ undefined

HKDSE 2008

32. 圖中，L_1、L_2、L_3 及 L_4 均爲直線。若 m_1、m_2、m_3 及 m_4 分別爲 L_1、L_2、L_3 及 L_4 的斜率，則下列何者必爲正確？



A. $m_1 < m_2 < m_3 < m_4$

B. $m_1 < m_2 < m_4 < m_3$

C. $m_2 < m_1 < m_3 < m_4$

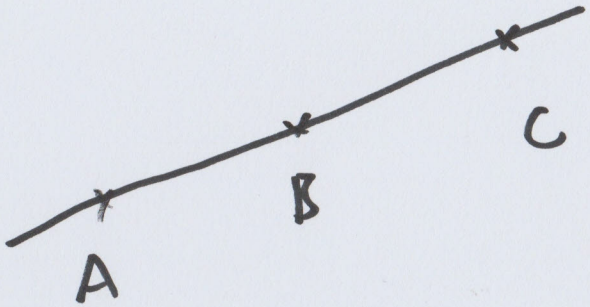
(D.) $m_2 < m_1 < m_4 < m_3$

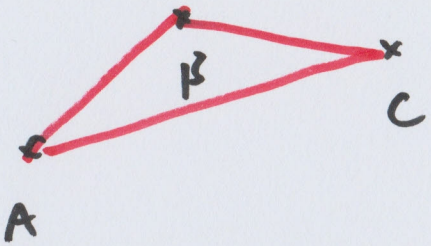
m_2 m_1 m_4 m_3

Applications of Slope

1.Collinear 共線	Test $m_{AB} = m_{BC}$
2.Parallel 平行	$m_1 = m_2$
3.Perpendicular 垂直	$m_1 \times m_2 = -1$

Collinear 共線

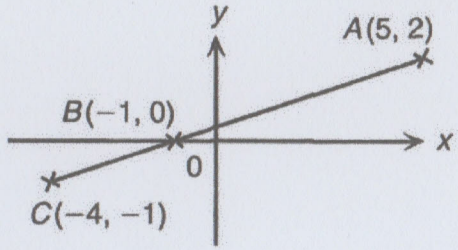




14. The figure shows three points $A(5, 2)$, $B(-1, 0)$ and $C(-4, -1)$.

(a) Find the slopes of AB and BC.

T7 **(b)** Determine whether the points A, B and C are collinear.



Source: Mathematics in Action (Second Edition) Supplementary Exercises 3B

(a) slope of $AB = \frac{2-0}{5+1} = \frac{2}{6} = \frac{1}{3}$

slope of $BC = \frac{0+1}{-1+4} = \frac{1}{3}$

(b) $\because$ slope of AB = slope of BC $= \frac{1}{3}$

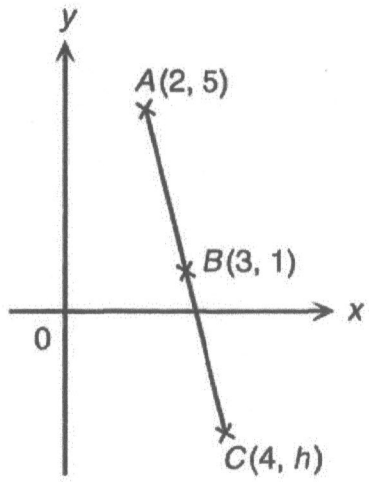
$\therefore$ They are collinear

Collinear 共線

15. In the figure, A, B and C are collinear.

(a) Find the slope of AB.

(b) Find the value of h.



Source: Mathematics in Action (Second Edition) Supplementary Exercises 3B

slope of $AB = \frac{5-1}{2-3} = \frac{4}{-1} = -4$

slope of $BC = \frac{h-1}{4-3}$

$\therefore$ A, B, C are collinear

$\therefore$ slope of AB = slope of BC

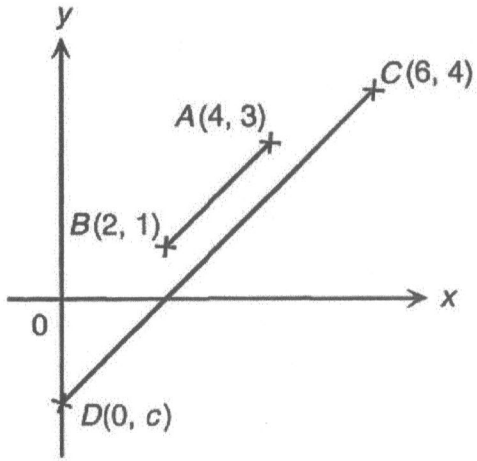
$$\frac{h-1}{4-3} = -4$$

$$h-1 = -4$$

$$h = -3$$

Parallel 平行 $m_1 = m_2$

15. In the figure, the line segment AB is parallel to another line segment CD. Find the value of c.



slope of AB = slope CD

$$\frac{3-1}{4-2} = \frac{4-c}{6-0}$$

Source: Mathematics in Action (Second Edition) Supplementary Exercises 3B

$$\frac{2}{2} = \frac{4-c}{6}$$

$$6 = 4 - c$$

$$c = 4 - 6$$

$$= -2$$

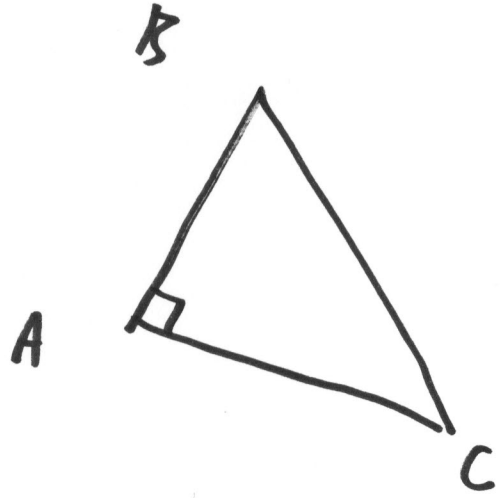
Perpendicular 垂直 $m_1 \times m_2 = -1$

13. It is given that $A(2, 3)$, $B(-3, 0)$ and $C(8, -7)$ are the vertices of $\triangle ABC$.

(a) Find the slopes of AB, BC and CA.

(b) Prove that $\triangle ABC$ is a right-angled triangle.

Source: Mathematics in Action (Second Edition) Supplementary Exercises 3B



slope of $AB = \frac{3-0}{2+3} = \frac{3}{5}$

slope of $BC = \frac{0+7}{-3-8} = \frac{7}{-11} = -\frac{7}{11}$

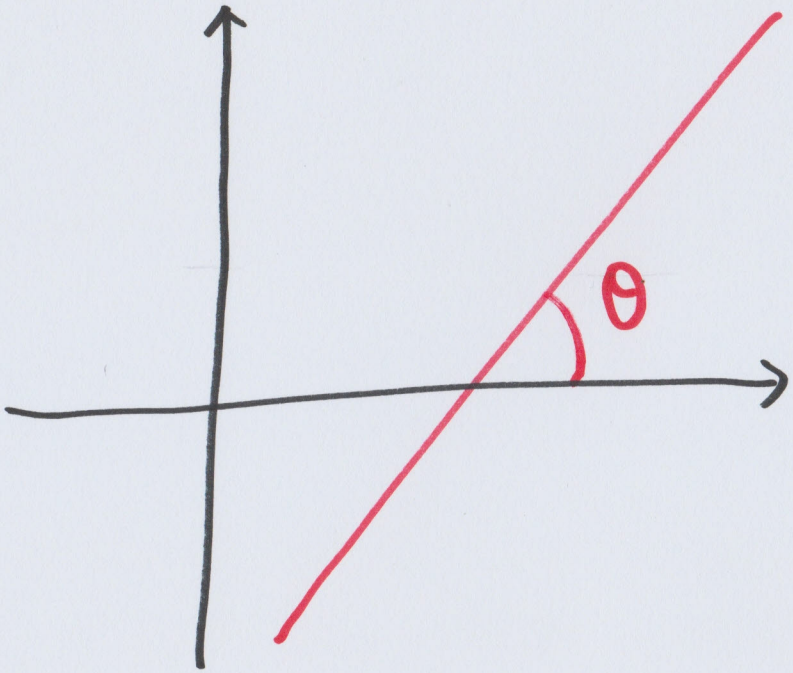
slope of $CA = \frac{3+7}{2-8} = \frac{10}{-6} = -\frac{5}{3}$

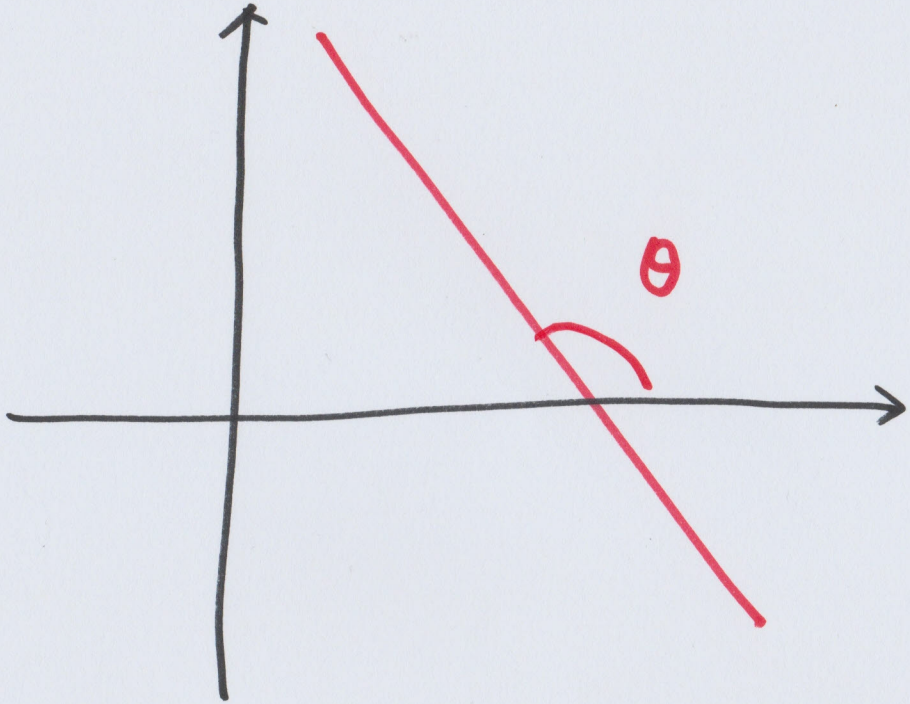
(b) slope of $AB \times$ slope of $CA = \frac{3}{5} \times \left(-\frac{5}{3}\right)$

$= -1$

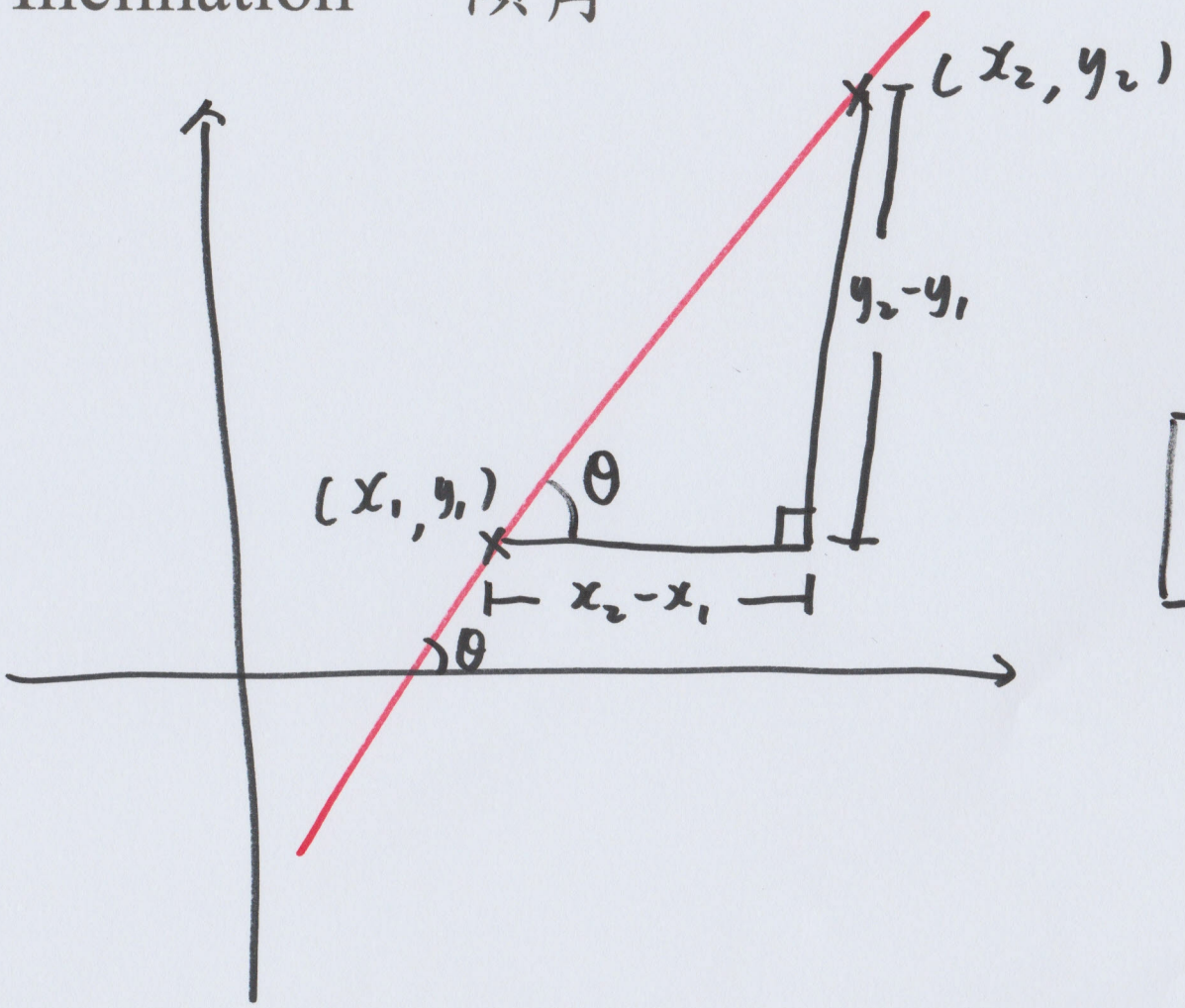
$\therefore AB \perp CA$

Inclination　傾角





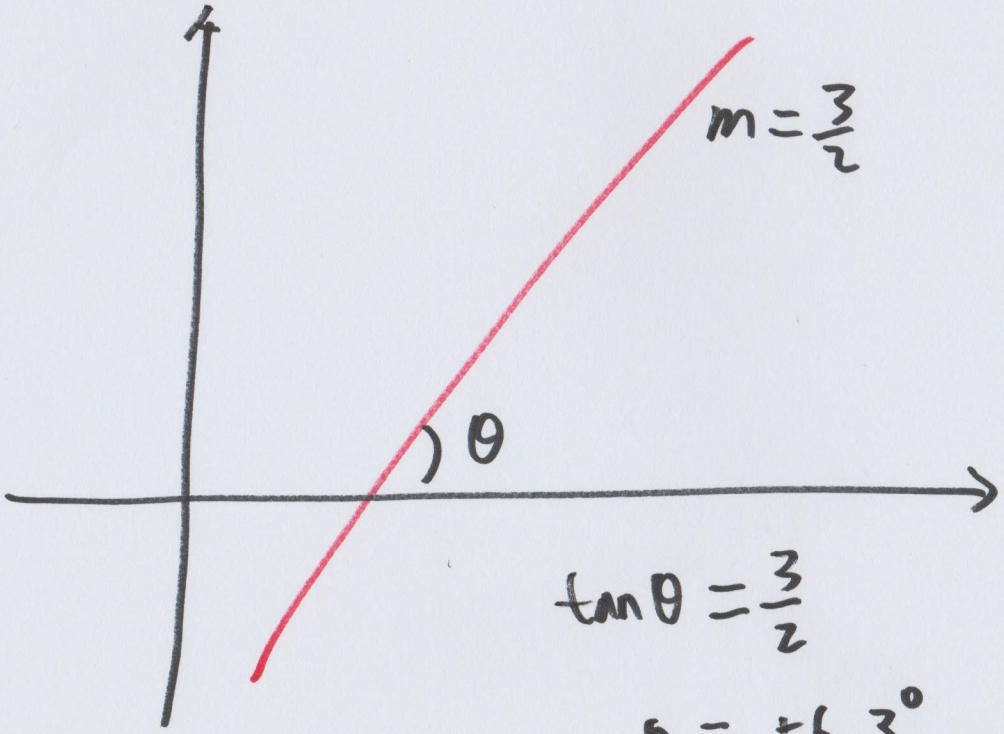
Inclination　傾角



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

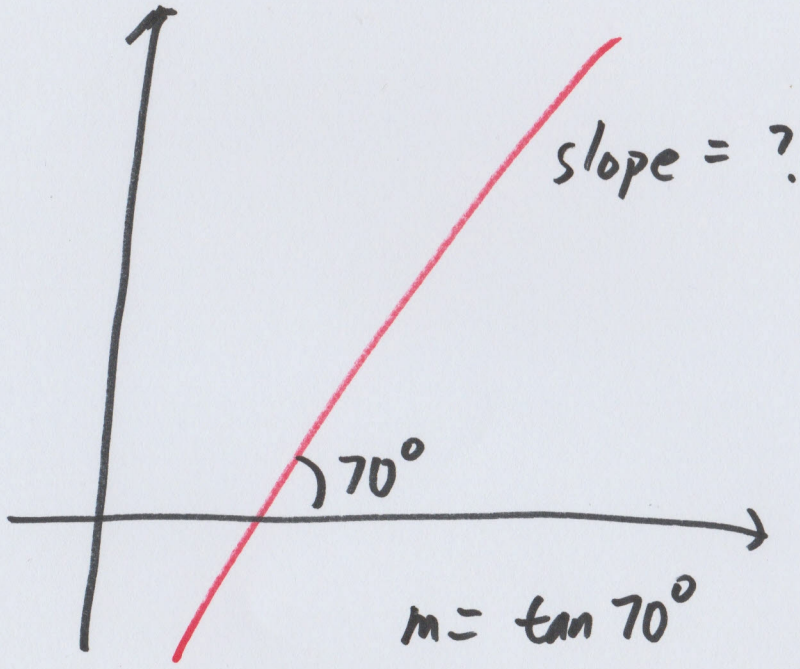
$$m = \tan\theta$$

Inclination　傾角



$\tan\theta = \frac{3}{2}$

$\theta = 56.3^\circ$



$m = \tan 70^\circ$

$= 2.75$

Inclination 傾角

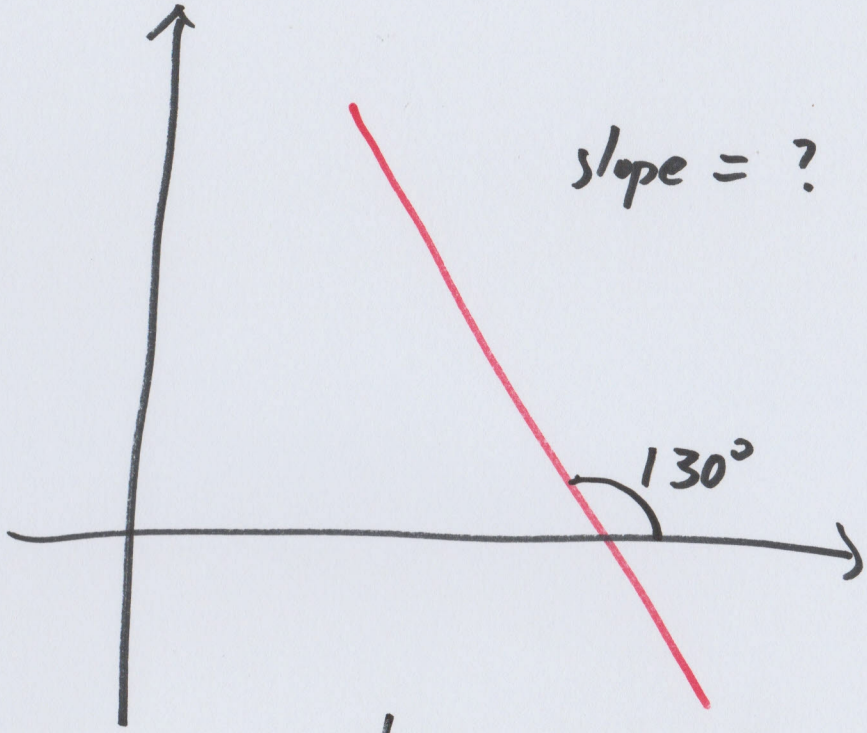
$m = \tan\theta$

$\tan\theta = -2$

From calculator

$\theta = -63.4^\circ + 180^\circ$

$= 116.6^\circ$

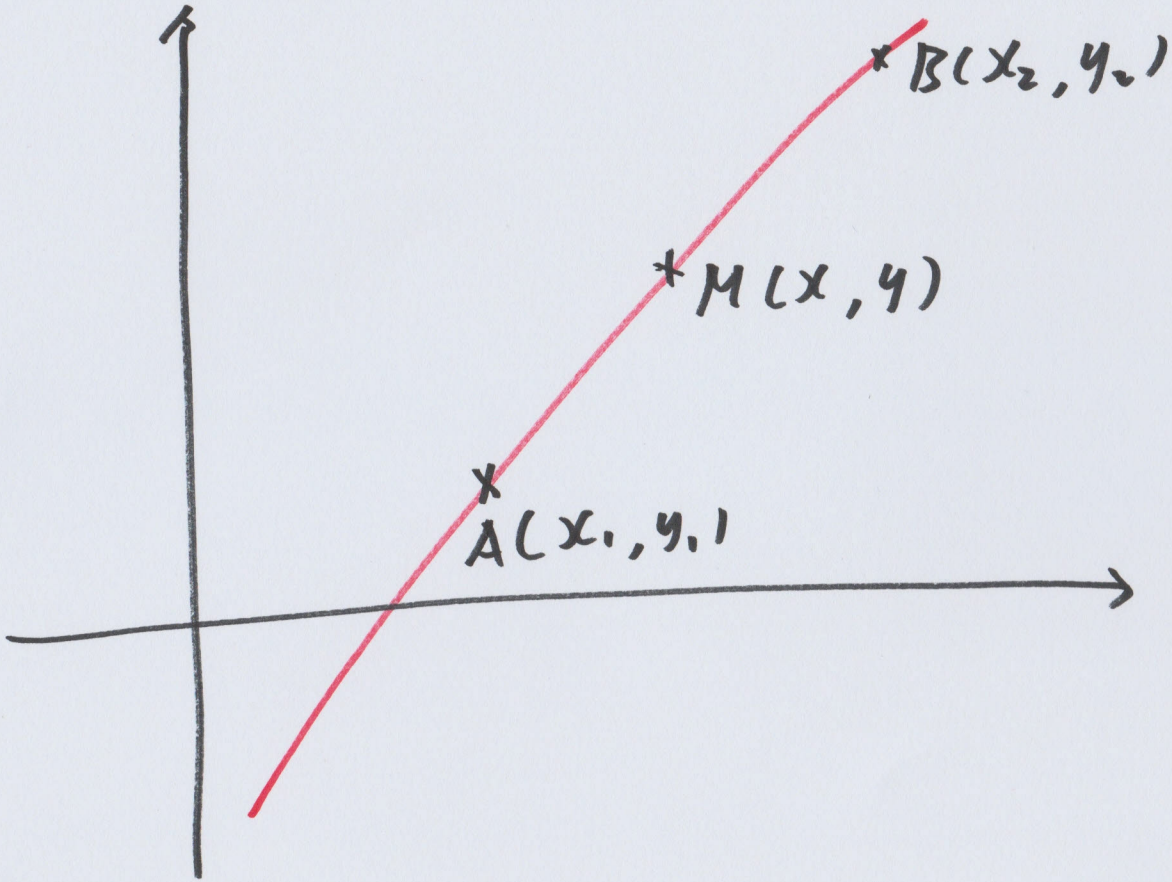


slope $= \tan\theta$

$m = \tan 130^\circ$

$= -1.19$

Mid-point Formula 中點公式

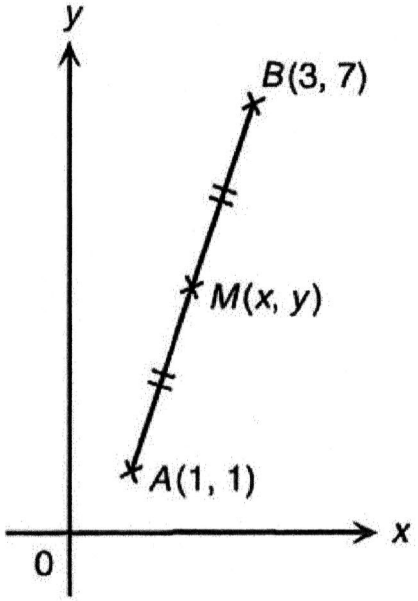


$$x = \frac{x_1 + x_2}{2}$$

$$y = \frac{y_1 + y_2}{2}$$

Mid-point Formula 中點公式

3.

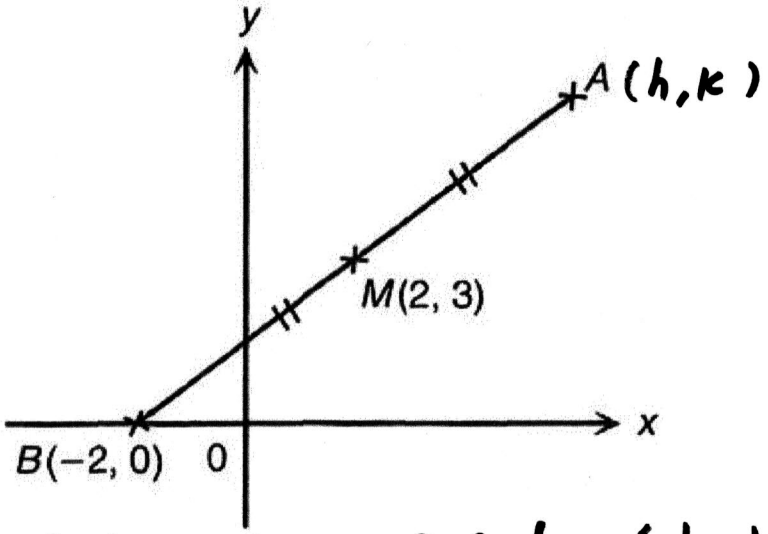


Source: Mathematics in Action (Second Edition) Supplementary Exercises 3B

$M = \left(\frac{1+3}{2}, \frac{1+7}{2}\right)$

$= (2, 4)$

14.



Let coordinates of A be (h, k)

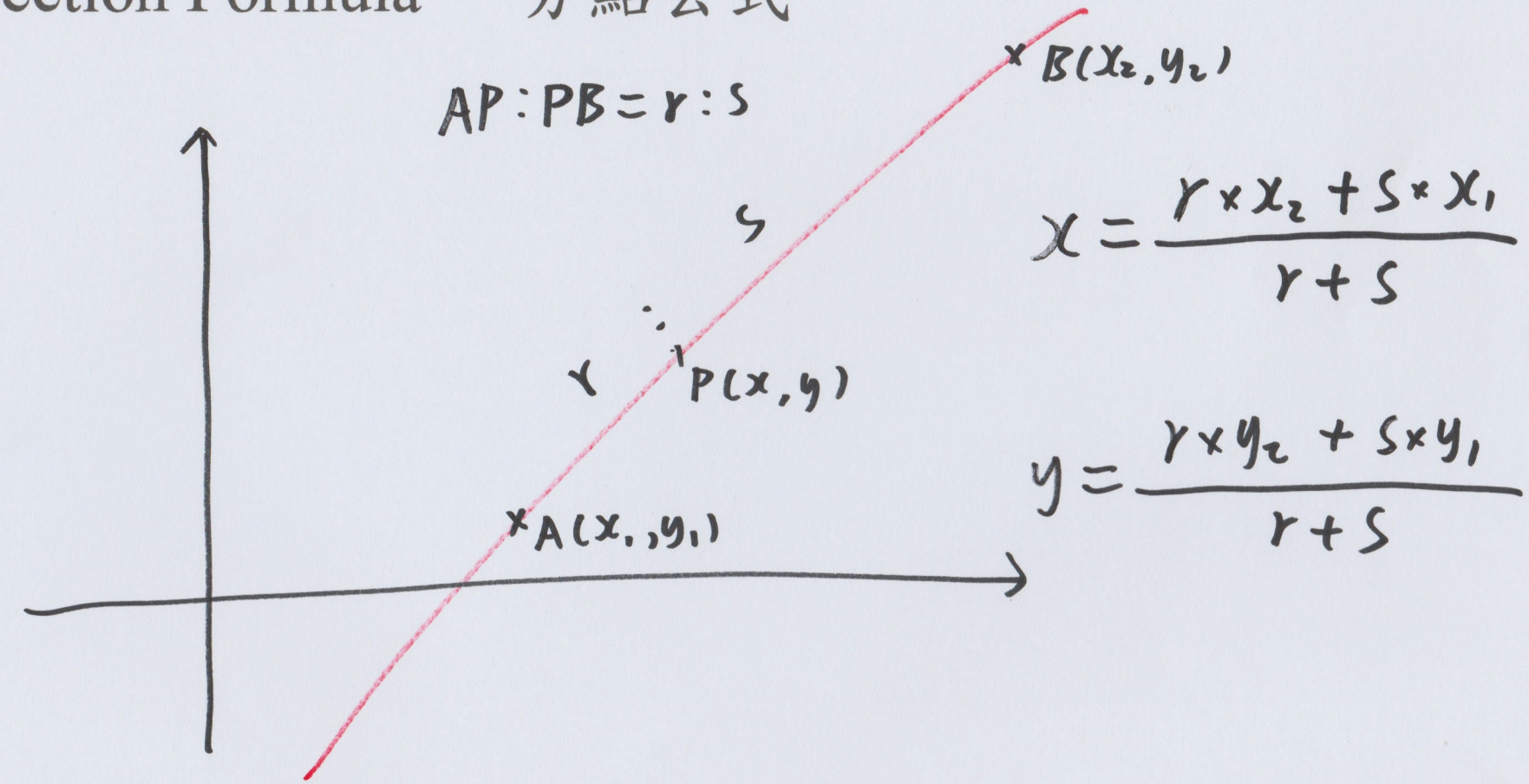
$\frac{-2+h}{2} = 2 \qquad \frac{0+k}{2} = 3$

$-2+h = 4 \qquad k = 6$

$h = 6$

$A = (6, 6)$

Section Formula 分點公式

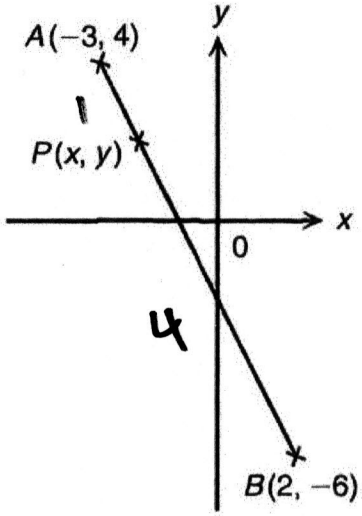


$$x = \frac{r \times x_2 + s \times x_1}{r + s}$$

$$y = \frac{r \times y_2 + s \times y_1}{r + s}$$

Section Formula 分點公式

10. $AP : PB = 1 : 4$



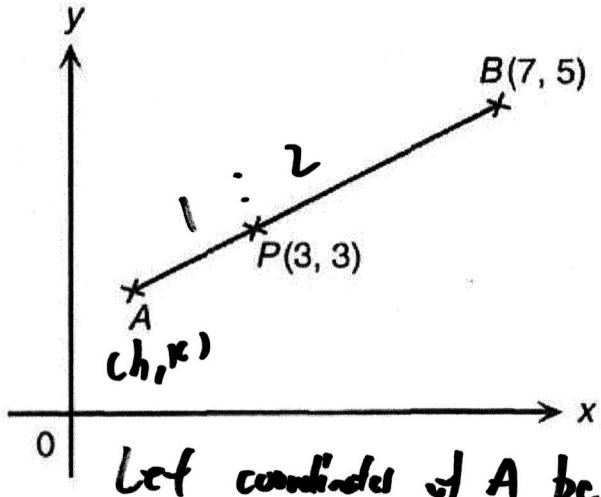
Source: Mathematics in Action (Second Edition) Supplementary Exercises 3B

$x = \frac{1\times 2 + 4\times(-3)}{1+4}$ $y = \frac{1\times(-6)+4\times 4}{1+4}$

$= -2$ $= 2$

$P = (-2, 2)$

17. $AP : PB = 1 : 2$



Let coordinates of A be (h, k)

$\frac{1\times 7 + 2\times h}{1+2} = 3$ $\frac{1\times 5 + 2k}{1+2} = 3$

$\frac{7+2h}{3} = 3$ $5 + 2k = 9$

$7 + 2h = 9$ $2k = 4$

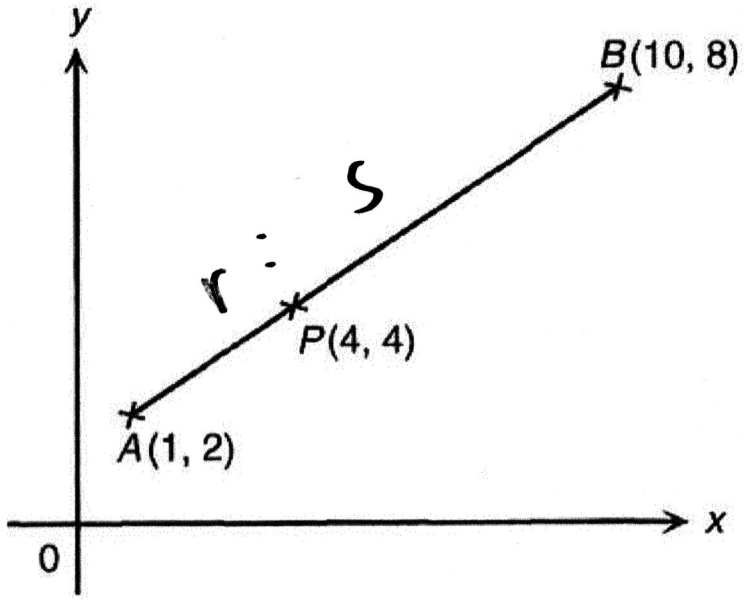
$2h = 2$ $k = 2$

$h = 1$

$A = (1, 2)$

Section Formula 分點公式

Find *AP*:*PB*



Source: Mathematics in Action (Second Edition) Supplementary Exercises 3B

Let $AP:PB = r:s$

$$\frac{10r+s}{r+s} = 4$$

$$10r+s = 4r+4s$$

$$6r = 3s$$

$$\frac{r}{s} = \frac{3}{6}$$

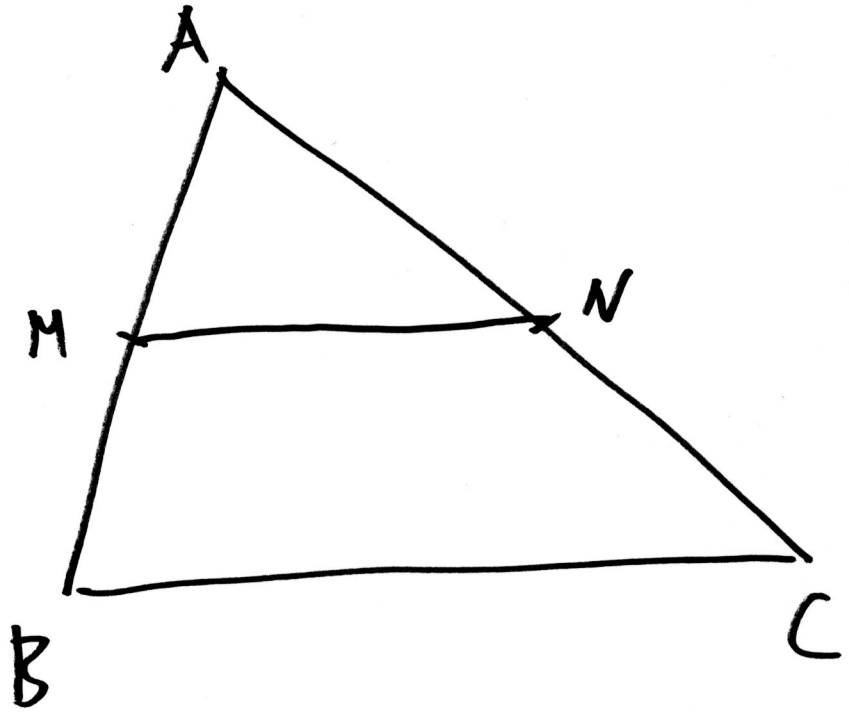
$$\frac{r}{s} = \frac{1}{2}$$

$$r:s = 1:2$$

$\therefore AP:PB = 1:2$ //

Analytic Approach 解析法

mid-point theorem 中點定理



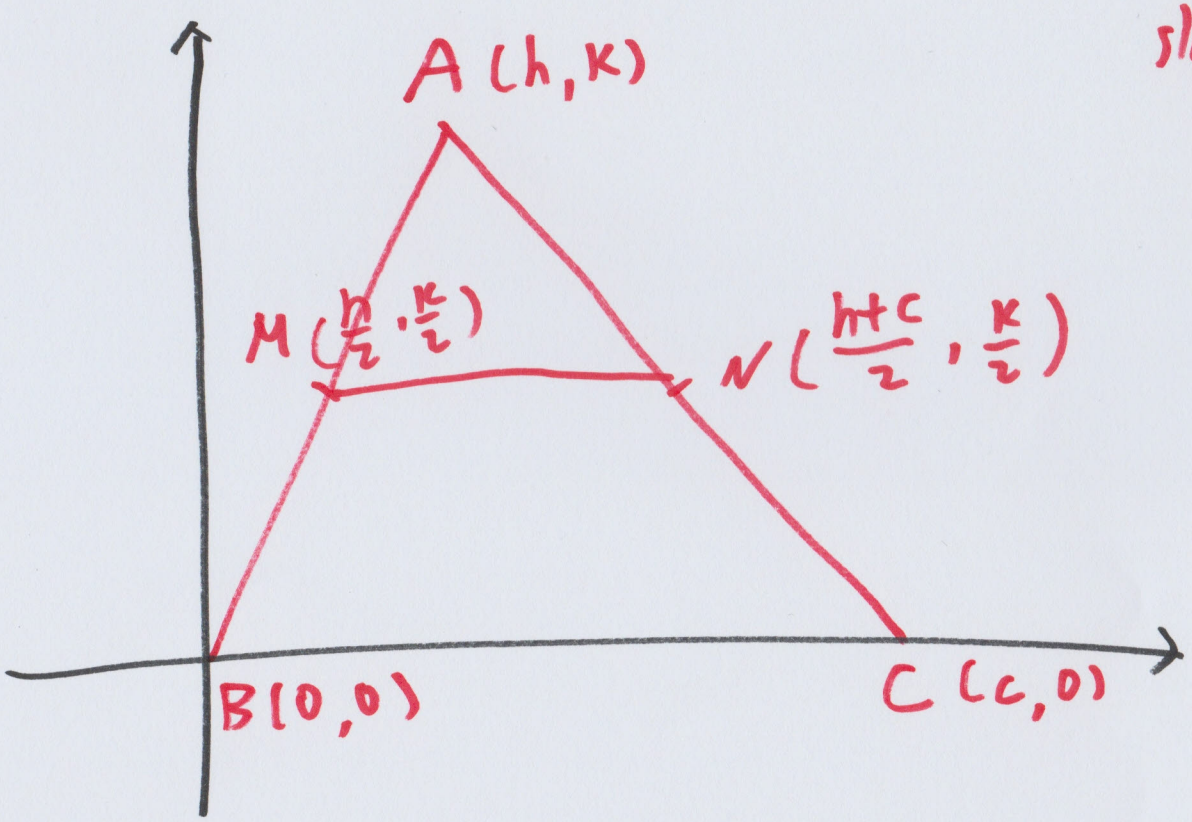
Given M and N are mid-points of AB and AC,

Then, $MN \parallel BC$ and

$$MN = \frac{1}{2}BC$$

Analytic Approach 解析法

mid-point theorem 中點定理



slope of $MN = 0$

slope of $BC = 0$

$\therefore MN \parallel BC$

$MN = \frac{h+c}{2} - \frac{h}{2} = \frac{c}{2}$

$BC = c$

$\therefore MN = \frac{1}{2} BC$

Analytic Approach 解析法

1.Distance Formula 距離公式

2.Slope 斜率

Parallel 平行, Perpendicular 垂直

$\tan \theta = m$ inclination 傾角

3. mid-point , section formula

中點公式，分點公式