

# Equation of Circles

Standard Form:  $(x - h)^2 + (y - k)^2 = r^2$       Centre at  $(h, k)$  with radius  $r$

General Form:  $x^2 + y^2 + Dx + Ey + F = 0$       Centre =  $\left( \frac{D}{-2}, \frac{E}{-2} \right)$

$$\text{Radius} = \sqrt{\left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - F}$$

## Finding the equation of a circle

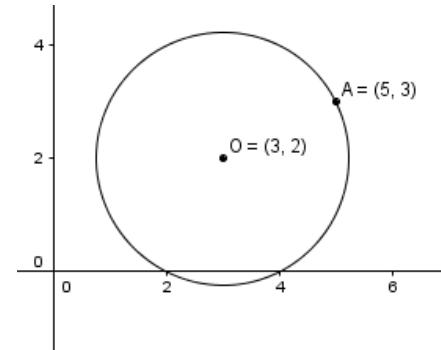
- 1) If Coordinates of the Centre and Radius are known, use Standard Form.

Example

$$\begin{aligned} r &= \sqrt{(5-3)^2 + (3-2)^2} \\ &= \sqrt{5} \quad (\text{keep it in surd form}) \end{aligned}$$

Equation:

$$\begin{aligned} (x-3)^2 + (y-2)^2 &= (\sqrt{5})^2 \\ (x-3)^2 + (y-2)^2 &= 5 \quad (\text{no need to expand LHS}) \end{aligned}$$



- 2) If three points on the circle are known, use General Form

Example

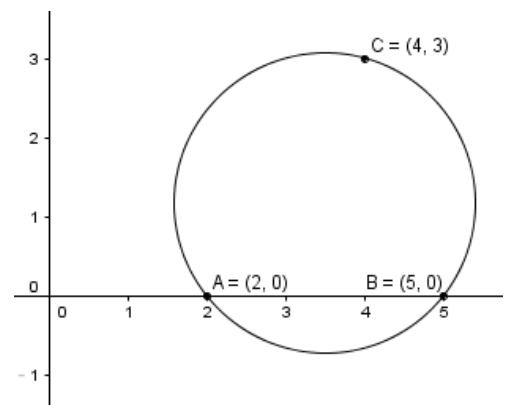
Let the equation be  $x^2 + y^2 + Dx + Ey + F = 0$ .

Put  $(2, 0)$ ,  $(5, 0)$  and  $(4, 3)$  into the equation.

$$\begin{cases} 2^2 + 0^2 + D(2) + E(0) + F = 0 \\ 5^2 + 0^2 + D(5) + E(0) + F = 0 \\ 4^2 + 3^2 + D(4) + E(3) + F = 0 \end{cases}$$

Solving the equations,  $D = -7$ ,  $E = \frac{-7}{3}$ ,  $F = 10$

Equation of circle is  $x^2 + y^2 - 7x - \frac{7}{3}y + 10 = 0$



# 圓形方程

標準式:  $(x - h)^2 + (y - k)^2 = r^2$  圓心在  $(h, k)$  及半徑為  $r$

一般式:  $x^2 + y^2 + Dx + Ey + F = 0$  圓心 =  $\left( \frac{D}{-2}, \frac{E}{-2} \right)$

$$\text{半徑} = \sqrt{\left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - F}$$

## 求圓形方程

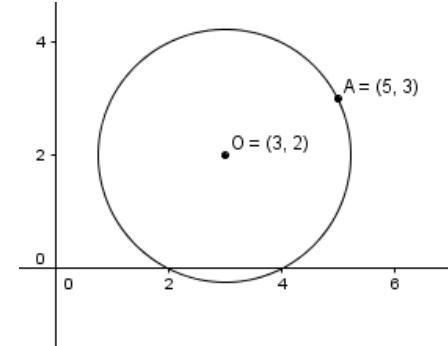
1) 如果已知圓心的坐標及半徑，運用標準式。

例子

$$\begin{aligned} r &= \sqrt{(5-3)^2 + (3-2)^2} \\ &= \sqrt{5} \quad (\text{保留以根式表示}) \end{aligned}$$

方程:

$$\begin{aligned} (x-3)^2 + (y-2)^2 &= (\sqrt{5})^2 \\ (x-3)^2 + (y-2)^2 &= 5 \quad (\text{無須展開 LHS}) \end{aligned}$$



2) 如果已知圓形上的三點坐標，運用一般式。

例子

設圓形方程為  $x^2 + y^2 + Dx + Ey + F = 0$ .

把  $(2, 0)$ 、 $(5, 0)$  及  $(4, 3)$  代入方程

$$\begin{cases} 2^2 + 0^2 + D(2) + E(0) + F = 0 \\ 5^2 + 0^2 + D(5) + E(0) + F = 0 \\ 4^2 + 3^2 + D(4) + E(3) + F = 0 \end{cases}$$

解聯立方程， $D = -7, E = \frac{-7}{3}, F = 10$

圓形的方程是  $x^2 + y^2 - 7x - \frac{7}{3}y + 10 = 0$

